Thresholding in Edge Detection

Yulka Petkova, Dimitar Tyanev
Technical University - Varna, Varna, Bulgaria

Abstract: *Many edge detectors are described in the literature about image processing, where the choice of input parameters are to be made by the users. In many cases such choices are made intuitively. In this paper we propose the choice of threshold to be based on the visual perception of the edge. Using our earlier edge definition and proposed algorithm for edge detection we formulate a mini-max rule for threshold determination. This rule uses the values of the second derivative of intensity function. Choice of the most significant values of second derivative allows to separate areas, which contain edges. The proposed approach is applied in the developed by the authors edge detector. The derived results are satisfactory.*

Keywords: *Edge Detection, Thresholding, Image Processing*

1. PROBLEM DEFINITION

The final decision in most issues of image processing that scientists have come up with is made when an appropriate solution is found. A mini-max rule is usually used and one of its aspects is a constant. In most of the cases the constant is chosen subjectively. The subjectivity stems from the fact that the constant expresses the appropriate extent of error that occurs when a problem is solved.

We are considering the following typical example in order to clarify our ideas. Let us find the value of the function *F(x)* which is to be computed as a sum of all elements of a descending order $F(x) = \sum_{i=0} e_i(x)$. According to the theory, if an order has an infinity number of elements, then

$$\lim_{i \to \infty} \left(\sum_i e_i(x) \right) \to F(x) .$$

This is the reason why in practice the computation of functions value is halted on condition that the before settled (acceptable) extent of error is reached. In order to summarize we will point out that the computation will be halted if the condition is satisfied. The rule (condition) is a kind of inequality. One of its parts is the subjectively chosen value that expresses the acceptable extent of error - ε. This statement we can formally express as follows:

(1) $$F_{i+1}(x) = F_i(x) + e_{i+1}(x) \quad \text{if} \quad | e_{i+1}(x) | \geq \varepsilon \, ; \quad i = 0,1,2,3, \ldots$$

The task for knowing a new image as an image which belongs to a particular class of objects Ω_i, i=1,2,3,… is achieved when it satisfies just that kind of rule, independently of its formal expression, for example:

(2) $$Image \begin{cases} \in \Omega_1 \, , & \text{if } \Delta(Image) < \varepsilon \, ; \\ \notin \Omega_1 \, , & \text{if } \Delta(Image) \geq \varepsilon \, . \end{cases} ,$$

where Δ*(Image)* defines the current value of the registered error in the image. This error is evaluated in the sense of any known criterion, which we do not comment here.

In the statement above we see a problem, which is usually passed over in silence. It is related to the choice of threshold, for which the value of the error can be accepted. The issue that we arise here is: "How is the threshold value computed?". Usually the answer is: "According to our (practical) experience we consider that the value … is satisfactory …" or "We can accept that it is correct …", or "Statistics show that …", etc.

We point out that we will try to figure out that threshold values are not formally computed. They are subjective and they are not universal. Their values vary depending of the result that is attended.

The specific task for edge detection in an image, that we are now discussing, arises inevitably an issue related to the examined case. It cannot be avoided. The problem was dearly stated at the end of [1] by asking the question: "What condition a particular point should satisfy in order to be flagged as an edge point?"

Edge detection is one of the most frequently used operations in image processing. The main reason for its widely spread usage is the fact that edges provide us with information about the structure of the image. Edge detection is a research area for more than 35 years [6, 7, 9, etc.]. Many registration techniques were developed. Each of them has advantages and drawbacks in terms of their implementation to a particular application. In the literature there are known several reviews of works about the edge detection. In order to make a summary we state that in each algorithm for edge detection a "measure" is subject to a particular edge registration. This measure is used in the process of edge registration. However authors do not explain the incentives for their decision. In most cases there is no explanation about the reason why a particular threshold was chosen and used. In most of the early edge detection algorithms the thresholds that are used are heuristically chosen. However results are not satisfactory as they depend on the type of the image that is sensed. Prewitt, Sobel and Roberts [2, 3, 4] operators and other that use zero-crossing of the second derivative of the intensity function use thresholds that are chosen without any precise motivation. The Canny detector [2, 3] is considered to be the most commonly used edge detector. In the Mathlab version of Canny detector is used a default threshold, which value is 75% of the gradient value. This value is considered to provide us with satisfactory results in most cases.

Far too many authors propose a solution in order to improve the outcome from the Canny detector. They stay that input parameters should be adaptively chosen [7]. Thus the usage of parameters with the fixed values is rejected.

Determination of the threshold values of the evaluation criterion in any edge detector is a specific task. The properly decision of this task is a precondition for deriving satisfactory results. In this paper we discuss the problem of thresholding in relation to the proposed by us edge detection algorithm [1].

2. FACTORS THAT HAVE AN INFLUENCE ON THE DETERMINATION OF THE THRESHOLD

In [1] we propose a new definition for an edge that is based on the discontinued first derivative of the intensity function. With a view to localizing discontinuance we suggest using the second derivative of intensity function. We state that most likely the position of the edge is located in the area around the local extremum of the second derivative of intensity as it is shown on the figure 1.

Firstly, we should point out that the absolute value of the extremum in the second derivative is in inversely proportional relation to the angle, which is formed when the two sides of the represented ideal edge cross each other. In addition to the statement above, we point out that the real edges are visually different from the type that is here presented and are inversely proportional to the mentioned angle - the acuter the angle is, the more easily the edge is distinguished. All of the above mentioned statements can be formulated in the following question: "What should be the measure of the angle, so that the edge will not be visualized?".

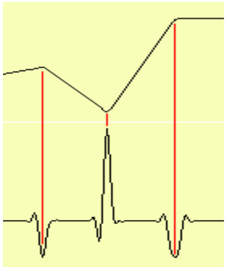

Fig.1. Intensity function and its second derivative

On the second place we should comment that the ideal edges are different from the real, which are rounded. The reasons about that are commented in [1]. The more rounded the angle is, the more undistinguished is the edge. As a result the elements of the image loss the "focus" and the quality of image decreases. The value of the second derivative is in inversely proportional relation to the roundness of the edge as well. If the roundness is high enough, e.g. it is not cause by the noise and apparatuses factors, it is considered to be a real one and the assumption of an edge is rejected. This issue arises a new question – "Which is the extent of roundness which will allow us state that there is an edge?".

In order to give an answer to the question on what condition a particular point belongs to an edge, we have to connect the value of estimation criterion with a particular constant in a minimax rule. The function that we can use according to the edge definition is the second derivative of the intensity function in the every point of image. Thus we are interested in and searching for the pick values of derivative, which bounded the areas of the most probably edges.

These pick values, as we have already mentioned above, depend on two factors:

1) the angle α between the sides of the edge and its position according to the axes x – angle β, see figure 2;
2) the extent of the edge rounding.

We want to connect the threshold value with the edge definition and with the visual effect as well. For instance, an ideal edge with the angle of 130^o between its sides is completely defined, because the first derivative of the intensity function is discontinued at the pick of the angle [1]. As a matter of fact the same angle cannot be visually distinguished and low values of the second derivative function belong to it. Shortly – high values of derivative correspond to the acute angles and low values of derivative correspond to the obtuse angles. Many images (real and artificial) were examined. The results inspired us to arrange a number of experiments in order to examine these factors.

During the first experiment we were engaged we research on the position of the angle according to the axes x. We consider two possible cases – if an angle is acute, its end positions are when one of its sides is perpendicular to the axes x, the second case is - if the angle is an obtuse one, its end positions are when one of its sides is parallel to the axes x – see figure 2.

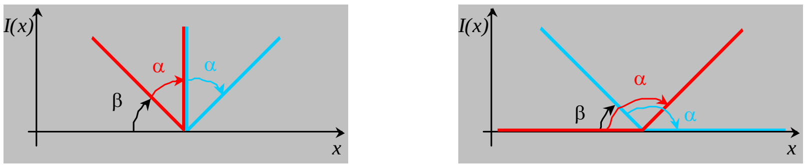

Fig.2. Left and right end position of the angle

Figure 3 shows the results of the first experiment. As it is seen, dependences are symmetrical and the highest values of derivative derive when the angles are in any of two end positions. When the angle is at the central position (its bisectrix is perpendicular to the axes x) the second derivative has got minimal value.

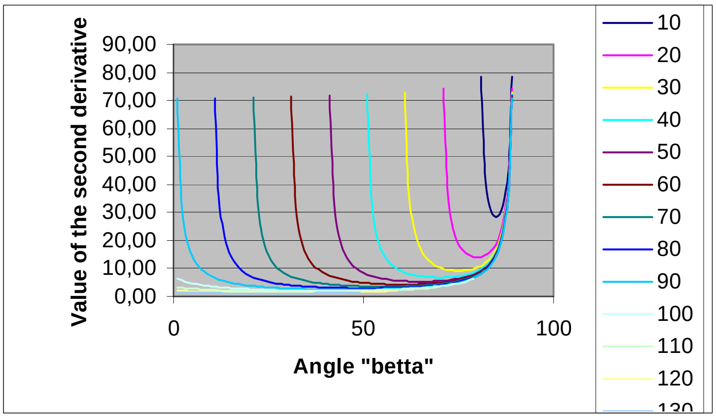

Fig. 3. Values of the second derivative against the angles α and β

Since the edge point determination is connected to the finding of pick values of derivative, the above said suggests to us that we have to examine the most unfavorable combination of the factors, which leads to the low values of derivative. With the other words we can reformulate the question – how low can be the value of the derivative, which corresponds to an edge point? Keeping in mind the second factor, we conclude that the most unfavorable angle position is the central one.

These are our motives for the second experiment – to examine the values of the second derivative of the intensity function, which corresponds to the angle in central position in relation to the different extent of its rounding – see figure 4.

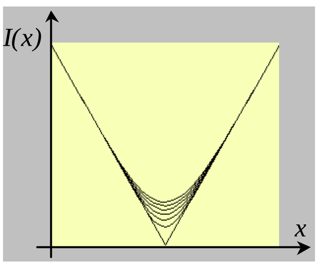

Fig.4. Angle with a different extent of rounding

We have registered the highest value of the derivative at the pick of the angle, together with the increasing extent of rounding, which is made by filtering. The derived family of curves is shown in the figure 5.

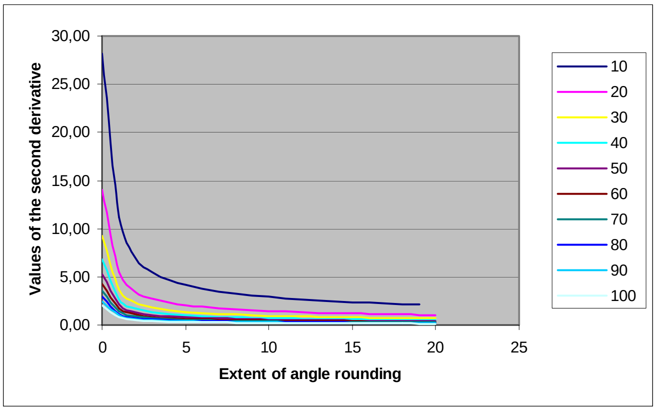

Fig. 5. Values of the second derivative depending
on the angle and the extent of its rounding

As it is seen, the values of derivative are highest when the angle is smallest and in the same time when it is less rounded.

Comparing plots from the figures 3 and 5 we can conclude, that the values of the second derivative for an angle, which is rounded, are equal to the values of the derivative for a greater, but not rounded angle. This ascertainment arouses an exclusive interest. If we consider it in the light of the second derivative function, for these two cases we will see two graphs with the same extreme values, and in the same time this one, which corresponds to the obtuse but not rounded angle has a smaller area, and in great extent is closely to the definition, i.e. to the Dirac δ-function. The function, which corresponds to the acute, but rounded angle, has the same extreme value, and in the same time it is wider, i.e. with the greater area. It is our profound conviction that these two parameters – the amplitude of derivative and its shape (area) can be integrated in a complex condition for determination of the threshold values. We are deeply believe, that their integration is possible only in the sense of our understanding for visual perception of the edge. Thus finally we arrive at a conclusion that it is necessary to make an experimental study of visual perception (distinguishing) of the edge. We are in the knowledge that in this experiment we do not be in a position to except the subjective factor. Because of that we consider, that the experiment should be carried out many times by the great number observers. Derived results we have to interpret statistically.

Such an experiment was carried out by the specially created for this purpose program with 72 observers. About 80% of them stop seeing edge when the angles are greater then 100^o.

Although we do not consider that this number is representative enough, to the present moment we can assume this value of the angle as a visual threshold for the edge perception. As we have already noted, we will bind threshold value with two factors – the extreme pick value of second derivative and its wide.

3. SUGGESTION FOR A MINIMAX RULE

Thus, we become to the moment when we have to synthesize the rule for determination the edge points in an image, which should integrate the mentioned above two factors. For the formal presentation of this rule we propose the following dependency:

$$\begin{aligned} &IF\ \left(\ddot{I}_{ij} > const1 \quad AND \quad W < const2\right) THEN \\ &\quad (i,j) \in Edge \\ &ELSE \\ &\quad (i,j) \notin Edge \end{aligned} \tag{3}$$

where: I_{ij} is the intensity at the point (pixel) *(i,j)* of the image;
$\ddot{I}_{ij}$ is the second derivative value at the point *(i,j)*;
Edge is a set of edge points;
const1 and *const2* are experimentally derived constants;
W [number of pixels] denotes the wide of derivative at the point *(i,j)*.

The rule (3) we have used in a program realization of the one of the proposed in [1] algorithms for edge detection in grey-scale images. The derived edges are shown in the figure 6.

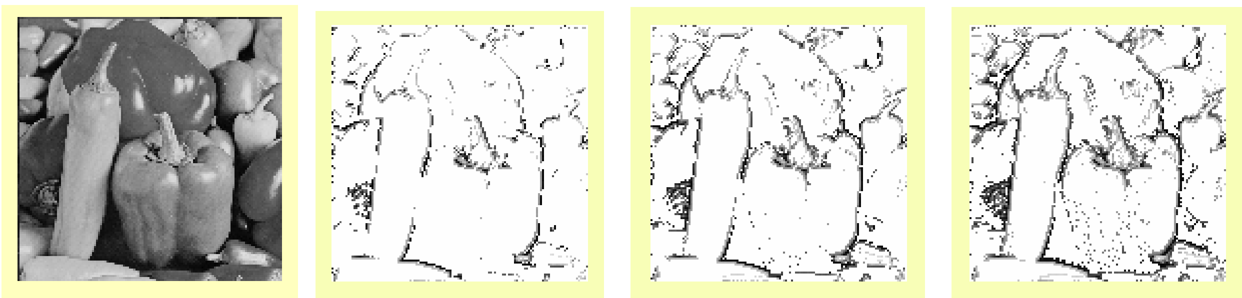

Fig. 6. Standard image "peppers" and derived edges

4. CONCLUSIONS

In the here presented study we have set out our understanding about the problem of thresholding in edge detection in the connection with the earlier published by us edge definition [1]. In the sense of this definition we have made by our opinion an in-depth analysis of the problem, some of its subjective metamorphoses, and we have chosen the possibly most logical basic point to the sense and determination the threshold constants. We have synthesized a formal mini-max rule for edge point detection and we have presented some preliminary results, derived with one of the possible algorithms, which are worked out by us.

5. REFERENCES

[1] Tyanev D.S., Y.P. Petkova, (2005), About the Possibilities of a New Edge Definition, 28th ISSE 2005, Vienna, Austria, IEEE Catalog Number 05EX1142C, ISBN: 0-7803-9325-2.

[2] Pratt W.K., (1991), Digital Image Processing, Wiley Computer Publishing, John Wiley&Sons, Inc.

[3] Ritter G.X., J.N.Wilson, (2001), Computer Vision Algorithms in Image Algebra, CRC Press

[4] Parker J.R., (1997), Algorithms for Image Processing and Computer Vision, Wiley Computer Publishing, John Wiley&Sons, Inc.

[5] Heath M.D., S. Sarkar, T. Sanocki, K.W. Bowyer, (1997), A Robust Visual Method for Assessing the Relative Performance of Edge Detection Algoithms, IEEE Transactions of Pattern Analysis and Machine Inteligence, vol. 19, pp. 1338-1359

[6] Heath M.D., S. Sarkar, T. Sanocki, K.W. Bowyer, (1998), Comparison of Edge Detectors: A Methodology and Initial Study, Computer Vision Image Understanding, vol. 69, pp. 38-54

[7] Rakesh R., P. Chudhuri, C. A. Murthy, (2004), Thresholding in Edge Detection: A Statistical Approach, IEEE Transactions on Image Processing, Vol. 13, No7, pp. 927-936

[8] Çokal, E., Özdemir, K., Alansal, A. and Erden, A., (1996), Development of a Vision Based System For Mobile Robots, 7th Int. Mach. Design and Production Conference, Turkey, pp. 595-604.

[9] Fu, K.S., Gonzalez, R.C. and Lee, (1987), C.S.G., Robotics, McGraw-Hill, p. 362.

[10] Sahoo P. K., Soltani S., (1988), A Survey of Thresholding Techniques, Computer Vision Graphics and Image Processing, 41, pp.233-260.